Segnale

26/09/2016

Def: grandezza fisica che trasporta un'informazione, è in funzione di qualche variabile $x(t)$ (variabile indipendente). È unidimensionale (dipende da una variabile)

$x(t)$: segnale; variabile indipendente: variazione segnale nel tempo

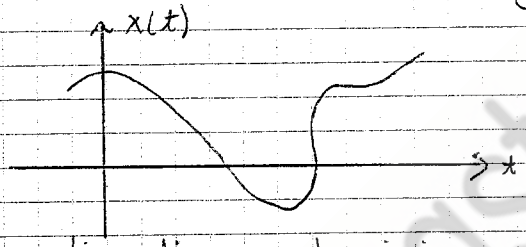
[Dominio: insieme di definizione della variabile indipendente / Codominio: " " del segnale stesso] Essi possono essere:

Discreto e Continuo (può assumere tutti i valori reali)

↓

può assumere solo alcuni valori discreti (es: t può assumere solo valori interi)

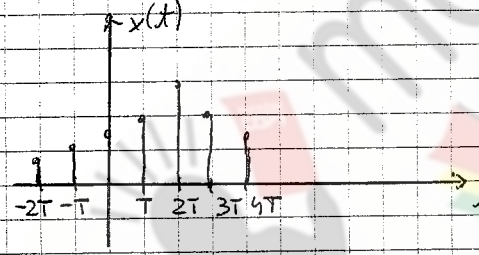
- Dominio e codominio continuo: segnali continui o analogici



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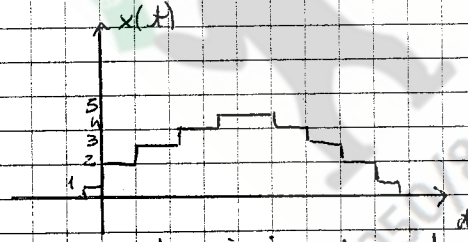
- Dominio discreto e codominio continuo: sequenza

t può assumere solo determinati valori

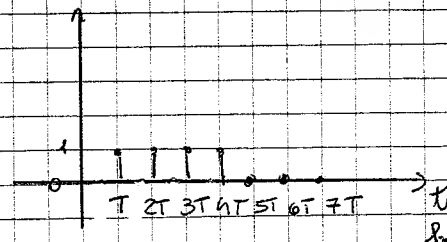


- Dominio continuo e codominio discreto: segnale quantizzato

il segnale può assumere solo determinati valori (livelli)



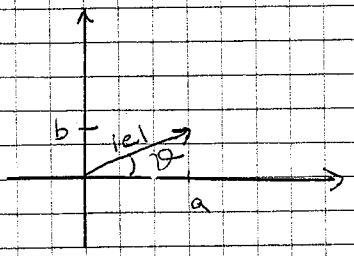
- Dominio e codominio discreto: segnale digitale



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$c = a + jb$ $\quad |c| = \sqrt{a^2+b^2}$ $\quad \angle c = \vartheta = \text{arc tg}\frac{b}{a}$ (fase) $\quad c = a + jb = |c| e^{j\vartheta}$

$e^{j\vartheta} = \cos\vartheta + j \sin\vartheta$



I segnali possono essere reali o complessi

- Segnali periodici: si ripetono periodicamente allo stesso modo

$x(t) = x(t + N T_0) \quad \forall t, \forall N$ intero con T_0 periodo

- Segnali aperiodici: non è possibile individuare un periodo per cui il segnale si ripete

Energia

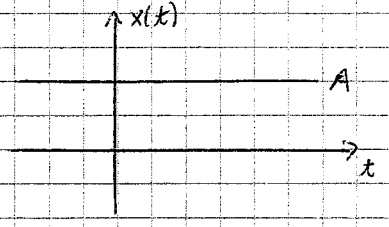
$x(t) \qquad E_x \triangleq \int_{-\infty}^{+\infty} |x(t)|^2 dt$

Potenza

$P_x \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$

Esempi:

$x(t) = A \qquad A = \text{cost}$



$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} A^2 dt = A^2 t \Big|_{-\infty}^{+\infty} = \infty$$

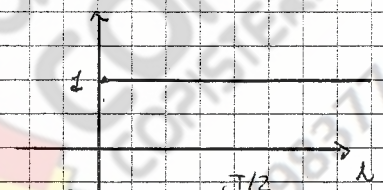
$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 dt = \lim_{T\to\infty} \frac{1}{T} A^2 t \Big|_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T} A^2 T = A^2 \quad \text{(potenza finita)}$$

I segnali si possono classificare per potenza e energia finita/infinita.

- Gradino unitario

$$u(t) = \begin{cases} 1 & \text{per } t > 0 \\ \frac{1}{2} & \text{per } t = 0 \\ 0 & \text{per } t < 0 \end{cases}$$



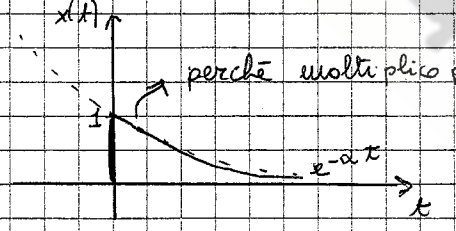
$$E_u = +\infty \qquad P_u = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} 1\, dt = \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T/2} 1\, dt = \frac{1}{T} \frac{T}{2} = \frac{1}{2}$$

Se ho potenza infinita, l'energia non può essere finita.

- Esponenziale monolatero (cioè $\neq 0$ solo da un lato)

$$x(t) = e^{-\alpha t} u(t) \quad \alpha > 0 = \begin{cases} e^{-\alpha t} & t \geq 0 \\ 0 & \text{altrove} \end{cases}$$

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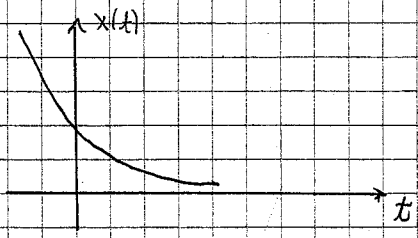


$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} e^{-2\alpha t} u(t)\, dt$$

$$= \int_{0}^{+\infty} e^{-2\alpha t} dt = -\frac{1}{2\alpha} e^{-2\alpha t} \Big|_{0}^{+\infty} = \frac{1}{2\alpha}$$

$$P_x = 0$$

- $x(t) = e^{-\alpha t}$



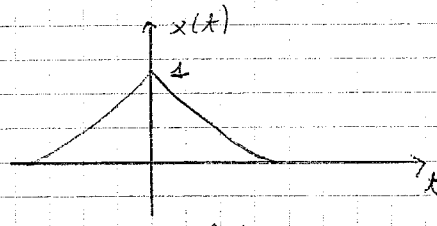
$$E_x = \int_{-\infty}^{+\infty} e^{-2\alpha t} dt = -\frac{1}{2\alpha} e^{-2\alpha t} \Big|_{-\infty}^{+\infty} = \infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-2\alpha t} dt =$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[-\frac{1}{2\alpha} e^{-2\alpha t} \right]_{-T/2}^{T/2}$$

$$= \lim_{T\to+\infty} -\frac{1}{2\alpha T}\left[e^{-\alpha T} - e^{\alpha T}\right] = \lim_{T\to\infty} \frac{e^{\alpha T}}{2\alpha T} = +\infty$$

$x(t) = e^{-\alpha|T|}$ esponenziale bilatero



$$E_x = \int_{-\infty}^{+\infty} e^{-2\alpha|T|}\,dt = \int_{-\infty}^{0} e^{-2\alpha|T|}\,dT + \int_{0}^{+\infty} e^{-2\alpha|T|}\,dT$$

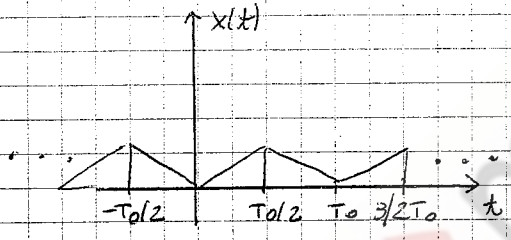
$$= 2\int_{0}^{+\infty} e^{-2\alpha t}\,dT = \frac{1}{\alpha}$$

$X(t) = A\,e^{j2\pi f_0 t} = A\cos(2\pi f_0 t) + jA\sin(2\pi f_0 T)$ $\quad A>0$

$E(x) = \int_{-\infty}^{+\infty} |x(t)|^2\,dt \qquad |x(t)| = |A|\,|e^{j2\pi f_0 t}| = A \cdot 1$

$$E(x) = \int_{-\infty}^{+\infty} A^2\,dt = +\infty$$

$x(t) = x(t + nT_0)$ $\qquad E(x) = \int_{-\infty}^{+\infty} |x(t)|^2\,dt = \lim_{N\to\infty} N\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt$



Energia segnali periodici $= \infty$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$$

$$= \lim_{N\to\infty} \frac{1}{NT_0} \propto \int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt$$

Potenza segnali periodici = finita

$x(t) = A\cos(2\pi f_0 t)$ segnale monocromatico, unica lunghezza d'onda

$x(t) = x(t + nT_0) = x\left(t + \frac{n}{f_0}\right) \qquad x\left(t + \frac{n}{f_0}\right) = A\cos\left(2\pi f_0\left(t + \frac{n}{f_0}\right)\right) =$

$= A\cos(2\pi f_0 t + 2\pi n) = A\cos(2\pi f_0 T)$ quindi $T_0 = \frac{1}{f_0}$ periodo $= T_0$

$f_0 = \frac{1}{2\pi}\frac{d\vartheta(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}(2\pi f_0 T) = f_0$ Hertz

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2\cos^2(2\pi f_0 t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2(1+\cos 4\pi f_0 t)}{2}\,dt$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2\,dt + \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2\cos 4\pi f_0 t}{2}\,dt = \frac{A^2}{2}$$

($\cos 4\pi f_0 t$ integrato su un periodo $= 0$)

$x(t) = A\sin(2\pi f_0 T + \vartheta) \qquad E_x = +\infty \qquad P_x = A^2/2$

$x(t) = A\,e^{j2\pi f_0 t}$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2\,dt = A^2$$

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- Segnali periodici a potenza finita 27/09/2016

Per segnali periodici P_x sul periodo $= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2\,dt < +\infty$ (cioè è finita)

$x(t) = A_0 + 2\sum_{K=1}^{+\infty} A_K \cos(2\pi K f_0 t + \vartheta_K)$ T_0 = periodo $f_0 = \frac{1}{T_0}$ = frequenza fondamentale

A_K = ampiezza, ϑ_K = fase, A_0 ⇓ componente continua, f_K = multiplo di f_0, per $K=0$ $f_K = 0$

$\cos x = \frac{e^{jx} + e^{-jx}}{2}$ sostituendo ottengo:

$$x(t) = A_0 + \sum_{K=1}^{+\infty} A_K \frac{e^{j(2\pi K f_0 t + \vartheta_K)} + e^{-j(2\pi K f_0 t + \vartheta_K)}}{2}$$

$$x(t) = A_0 + \sum_{K=1}^{+\infty} A_K e^{j\vartheta_K} e^{j2\pi K f_0 t} + \sum_{K=1}^{+\infty} A_K e^{-j\vartheta_K} e^{-j2\pi K f_0 t}$$

cambio indice $K = -K$

$$\sum_{K=-\infty}^{-1} A_{-K} e^{-j\vartheta_{-K}} e^{j2\pi K f_0 t}$$

pongo $A_0 = x_0$ per $\begin{cases} K>0 & x_K = A_K e^{j\vartheta_K} \\ K<0 & x_K = A_{-K} e^{-j\vartheta_{-K}} \end{cases}$ e ottengo:

Formule di Sintesi

$$x(t) = \sum_{K=-\infty}^{+\infty} x_K e^{j2\pi K f_0 t}$$ supponiamo che converge a $x(t)$

$$\int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi n f_0 t}\,dt = \int_{-T_0/2}^{T_0/2} \left[\sum_{K=-\infty}^{+\infty} x_K e^{j2\pi K f_0 t}\right] e^{-j2\pi n f_0 t}\,dt$$

$$= \int_{-T_0/2}^{T_0/2} \sum_{K=-\infty}^{+\infty} x_K e^{j2\pi(K-n) f_0 t}\,dt = \sum_{K=-\infty}^{+\infty} \int_{-T_0/2}^{T_0/2} x_K e^{j2\pi(K-n) f_0 t}\,dt$$

$$= \int_{-T_0/2}^{T_0/2} x_K e^{j2\pi(K-n) f_0 t}\,dt = x_K \int_{-T_0/2}^{T_0/2} e^{j2\pi(K-n) f_0 t}\,dt =$$

$$= \frac{e^{j2\pi(K-n) f_0 t}}{j2\pi(K-n) f_0}\Bigg|_{-T_0/2}^{T_0/2} = \frac{e^{j\pi(K-n)} - e^{-j\pi(K-n)}}{j2\pi(K-n) f_0}$$

$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$ ottengo: $\frac{\sin \pi(K-n)}{\pi(K-n) f_0}$ $K \neq n \in N$ il risultato è zero

per $K = n$ $\int_{-T_0/2}^{T_0/2} 1\,dt = T_0$

$$\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt = T_0 X_n \qquad X_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-j2\pi n f_0 t}\, dt$$

Formula di analisi
(coefficienti serie di Fourier)

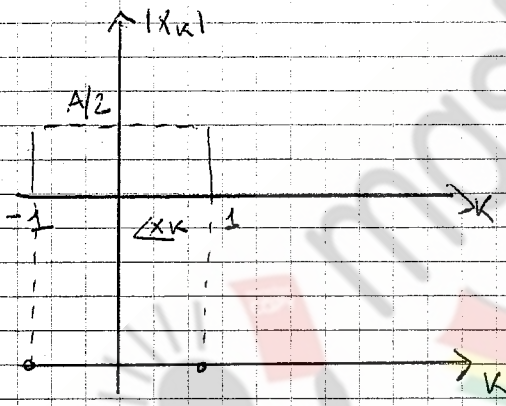
$x(t) = A\cos(2\pi f_0 t)$ con $A>0$ segnale periodico di periodo T_0 $\quad P_x = \frac{A^2}{2}$

$$X_K = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A\cos(2\pi f_0 t)\, e^{-j2\pi K f_0 t}\, dt$$

($K=1$, $K=-1$)

$$x(t) = \frac{A}{2} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j2\pi f_0 t}$$

$X_K = \frac{A}{2}$ per $K = \pm 1$ $\qquad X_K = 0$ altrove $\qquad$ i coefficienti della fase di Fourier si chiamano spettri



$$x(t) = A\sin(2\pi f_0 t) = \frac{A}{2j} e^{j2\pi f_0 t} - \frac{A}{2j} e^{-j2\pi f_0 t}$$

$X_K = \begin{cases} K=1 & \frac{A}{2j} \\ K=-1 & -\frac{A}{2j} \\ K=0 & 0 \end{cases}$

